

Vertical Alignment



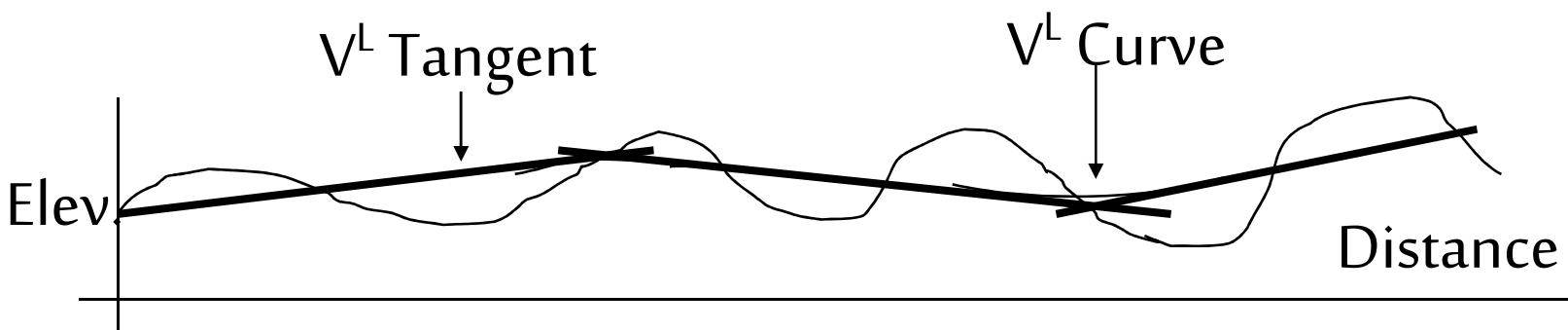
Vertical Alignment

In vertical alignment, the following will be discussed:

1. Overview,
2. Max. and min. grades,
3. Grades length (Critical length of grades),
4. Climbing lanes,
5. Types and basic design elements,
6. Elevations and stations on vertical curves,
7. Sight distance,

Vertical Alignment

Vertical Alignment is a basics of vertical tangents connected by vertical curves.



- Specifies the elevation of points along a roadway
- Provides a transition between two grades
- Sag curves and crest curves
- Equal-tangent curves - half the curve length positioned before the PVI; half after

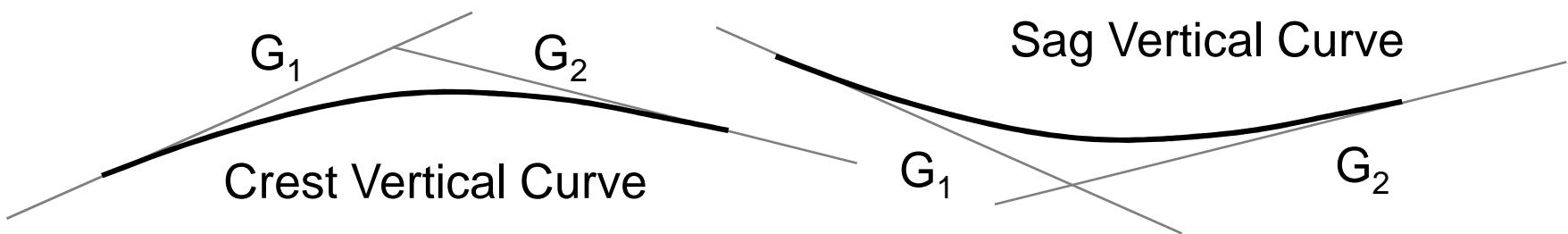
Objective of Vertical Alignment

Determine elevation to ensure

- Proper drainage
- Acceptable level of safety

The primary challenge

- Transition between two grades using two types of vertical curves (Sag curves & Crest curves)



Max. Grade

Max. grade depends on the following parameters:

Design speed, Design vehicle, Type of topography

Three types of topography (level, rolling, and mountainian)

The first number for the level topography whereas, the second number for the mountainian topography for each type of highway.

Design speed	Freeway	R. Arterial	R. Collector	U. Collector
<50				8- 14
50			7-9	7-11
60			6-8	6-9
70		4-6	5-7	
80		4-5	4-6	
90	3-5	3-5		
100	3-5	3-5		
>100	3-4	---		

Recommended Maximum Grades

Highway	Design speed (mph)	Maximum Grades (%)		
		Type of Terrain		
		Level	Rolling	Mountainous
Urban collectors	20	9	12	14
	30	9	11	12
	40	9	10	12
	50	7	8	10
	60	6	7	9
	70	5	6	7
Rural Arterials	50	4	5	7
	60	3	4	6
	70	3	4	5
Freeways	50	4	5	6
	60	3	4	6
	70	3	4	5

Min. grades rang from **0.3%** to **0.5%** to provide good pavement drainage

Critical Length of Grade

The critical length of grade is the maximum length of an upgrade on which a loaded truck can operate without an unreasonable reduction in speed. A reduction in speed of 15 kph suggested by AASHTO as a design criteria guide for determining the critical length of grade. If the physical controls or the nature of terrain cause a speed reduction greater than that the suggested design, it desirable to provide a **climbing lane** as an extra lane on the highway where the critical length of the grade is exceeded.

Critical length of grade depends on :

- Initial speed (at the beginning of grade)
- Value of upgrade
- Truck characteristics (180kg /hp ratio)

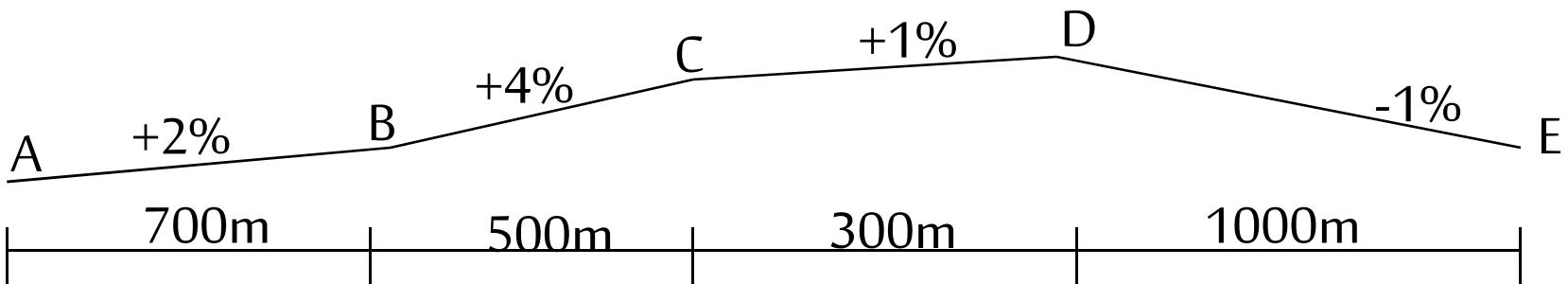


Climbing Lane

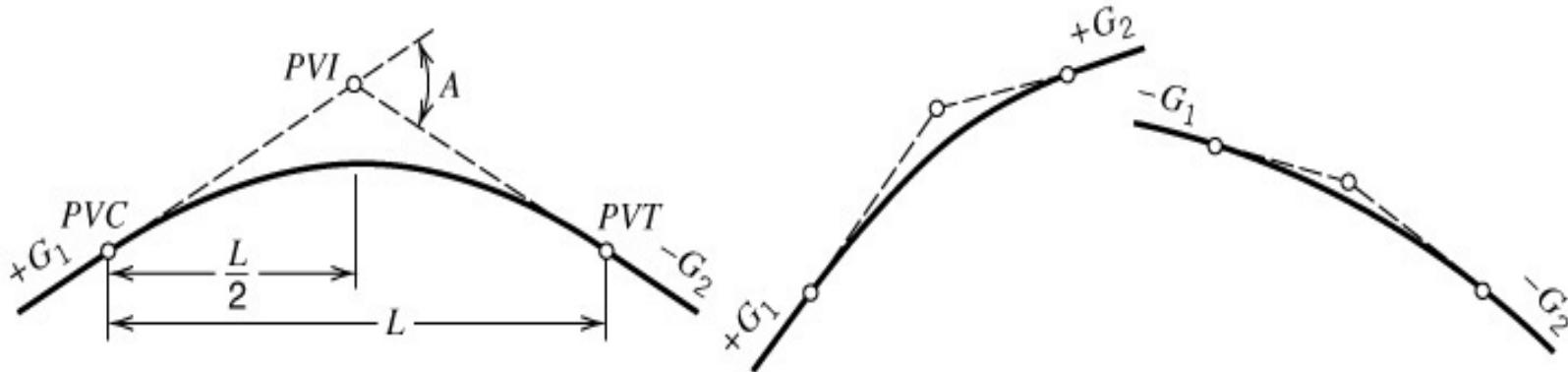
- A climbing lane is an added uphill lane for use by slow-moving vehicles whose speeds are reduced because of grade.
- The climbing lane is provided only if the volume of traffic is at or near capacity and the truck volume is high in addition to the critical length requirements.
- The climbing lane are more common for 2-lane highway than multi-lanes highway.
- The climbing lanes are design for each direction independent of the other.

Examples

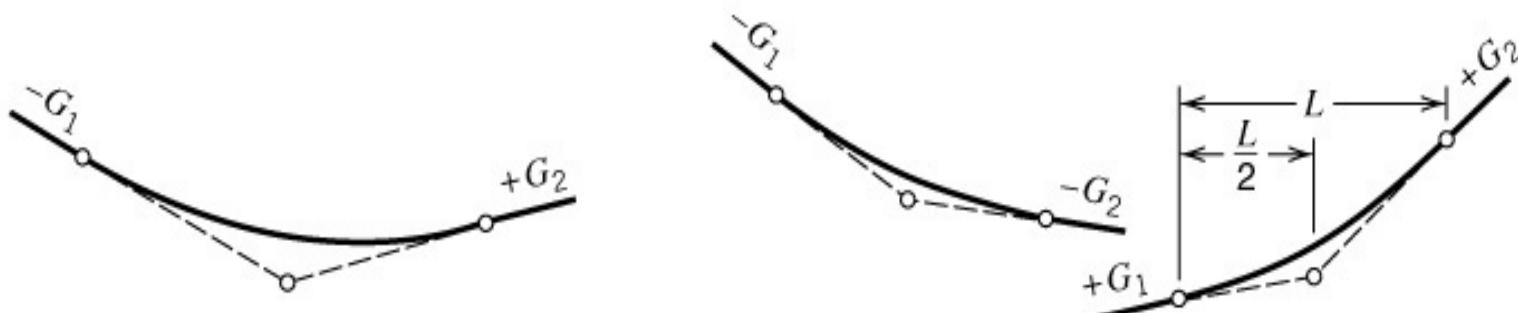
Given a profile section of 2-lanes highway in the following figure, the average speed is 90kph. Neglect the presence of vertical curves, determine the speed at the end point E of this section. Is it required to have a climbing lane on this section? if yes, determine its location and its total length.



Sag Curves and Crest Curves



$$A = G_1 - G_2 = + \quad \text{Crest Vertical Curves}$$



$$A = G_1 - G_2 = - \quad \text{Sag Vertical Curves}$$

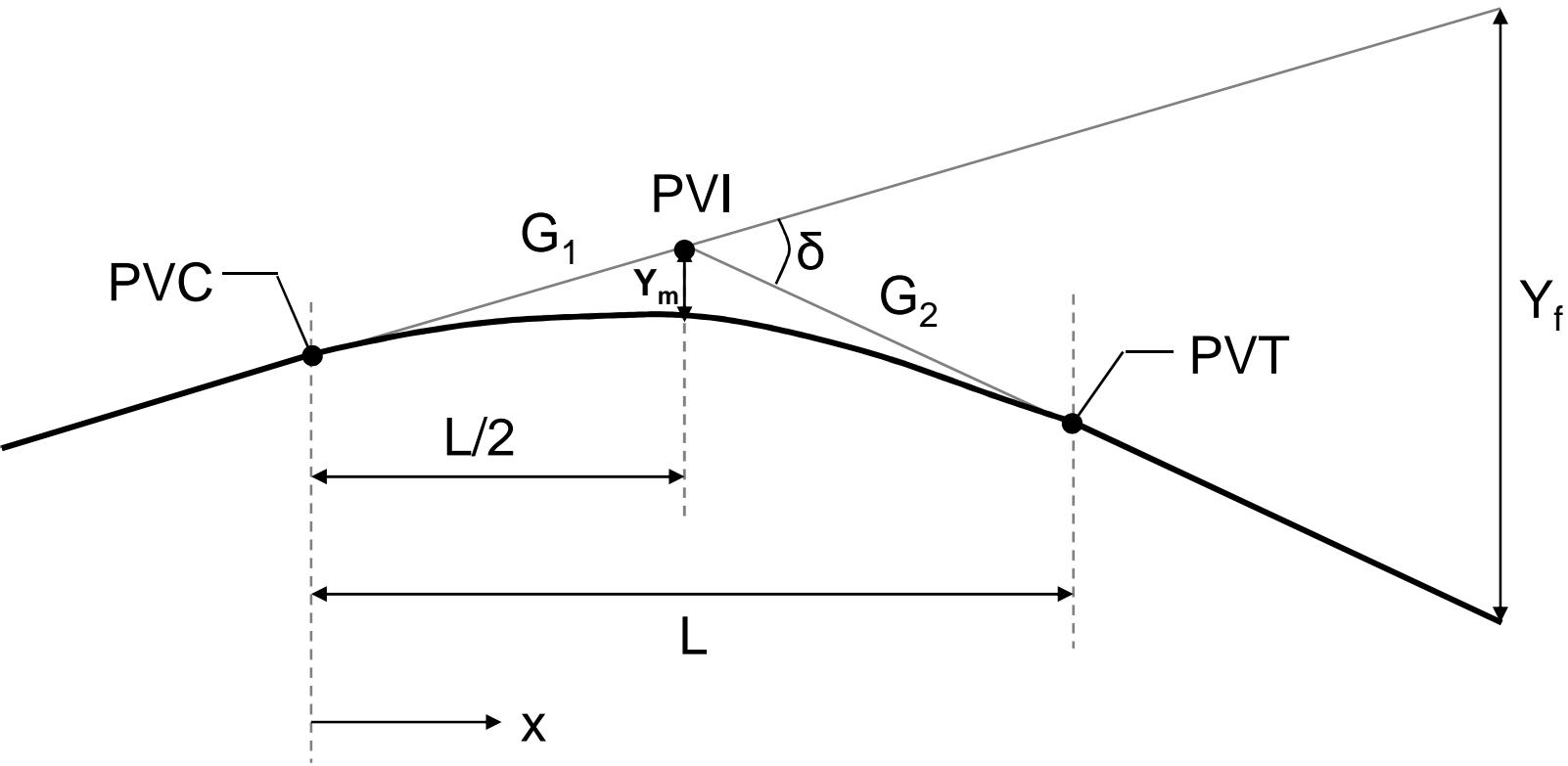
Vertical Curve Fundamentals

- **Parabolic function**
 - Constant rate of change of slope
 - Implies equal curve tangents

$$y = ax^2 + bx + c$$

- **y is the roadway elevation x stations from the beginning of the curve**

Vertical Curve Fundamentals



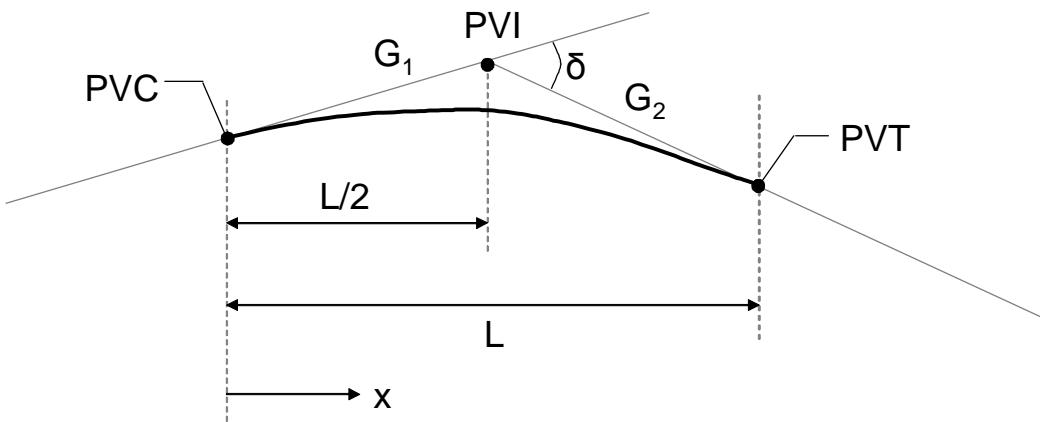
$$y = ax^2 + bx + c$$

Relationships

At the PVC: $x = 0$ and $Y = c$

At the PVC: $x = 0$ and $\frac{dY}{dx} = b = G_1$

Any where: $\frac{d^2Y}{dx^2} = 2a = \frac{G_2 - G_1}{L} \Rightarrow a = \frac{G_2 - G_1}{2L}$



Vertical Alignment Relationships

$$y = ax^2 + bx + c$$

$$\frac{dy}{dx} = 2ax + b$$

at PVC, $x = 0$:

$$b = \frac{dy}{dx} = G_1$$

$$\frac{d^2 y}{dx^2} = 2a$$

$$a = \frac{G_2 - G_1}{2L}$$

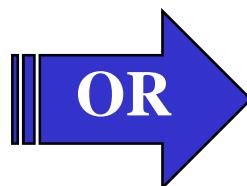
$$Y = \frac{A}{200L} x^2$$

$$Y_m = \frac{AL}{800}$$

$$Y_f = \frac{AL}{200}$$

$$K = \frac{L}{A}$$

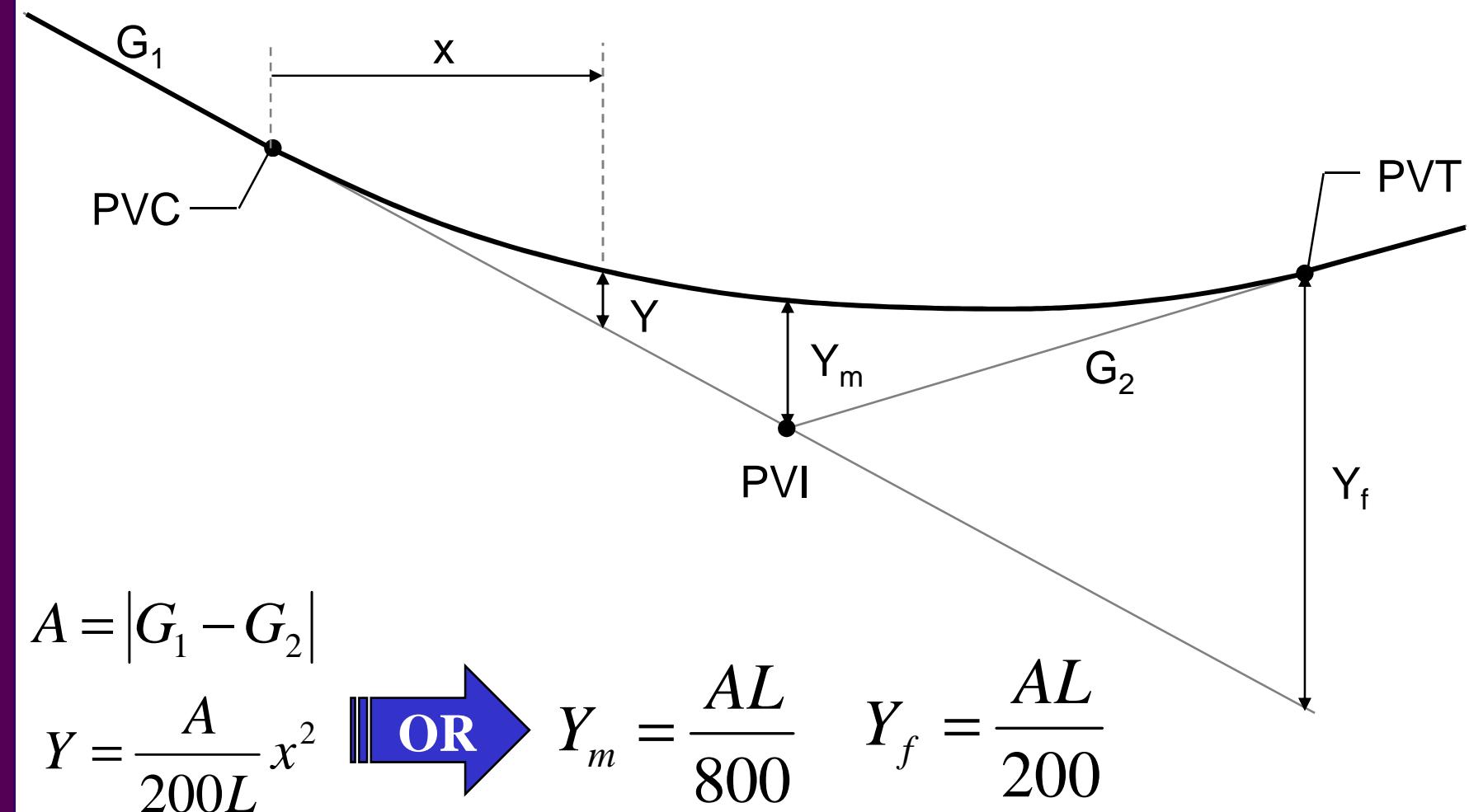
The offset (y) at
distance (X) from (PVC)



$$Y = \frac{A}{200L} x^2$$

$$A = |G_1 - G_2|$$

Other Properties



Critical point

Critical point is the turning point on the vertical curve (i.e. it is the highest point on the crest curve and the lowest point on sag curve), it lies at $dy/dx=0$ for the curve equation (**Note that G and A in decimal**)

$$y = ax^2 + bx + c$$

$$\frac{dy}{dx} = 2ax + b$$

at PVC , $x = 0$:

$$b = \frac{dy}{dx} = G_1$$

$$\frac{d^2y}{dx^2} = 2a$$

$$a = \frac{G_2 - G_1}{2L}$$

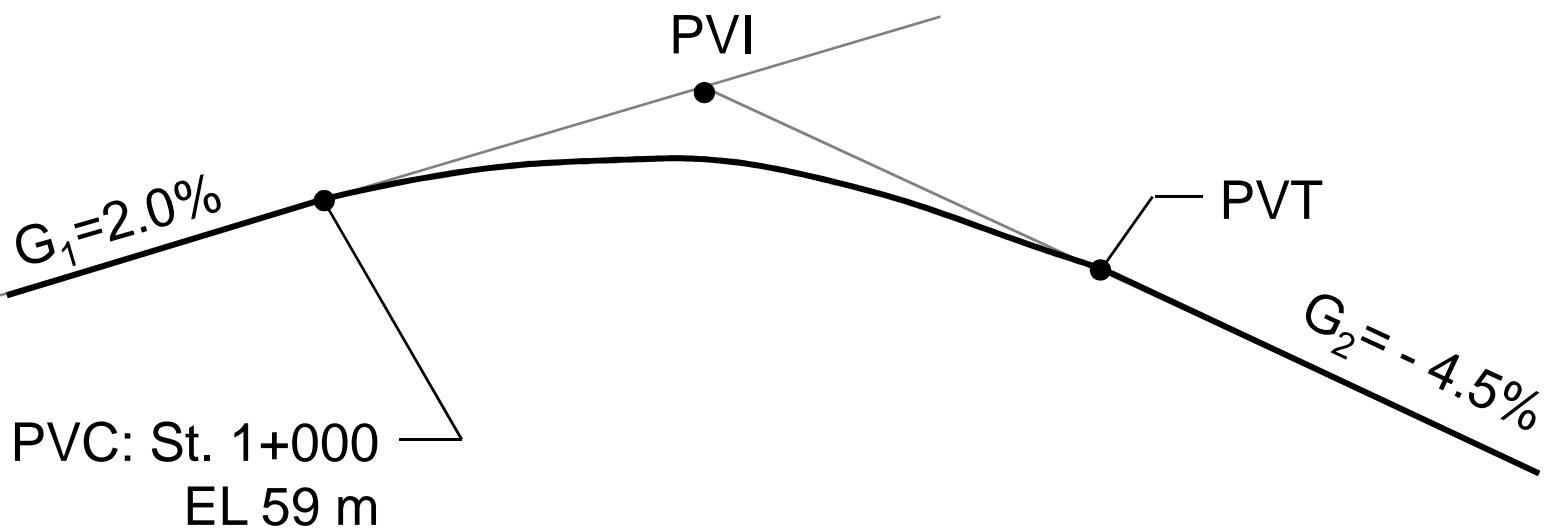
or using the relations

$$X_{cr} = \frac{g_1}{A} L$$

$$Y_{cr} = \frac{A(X_{cr})^2}{200L}$$

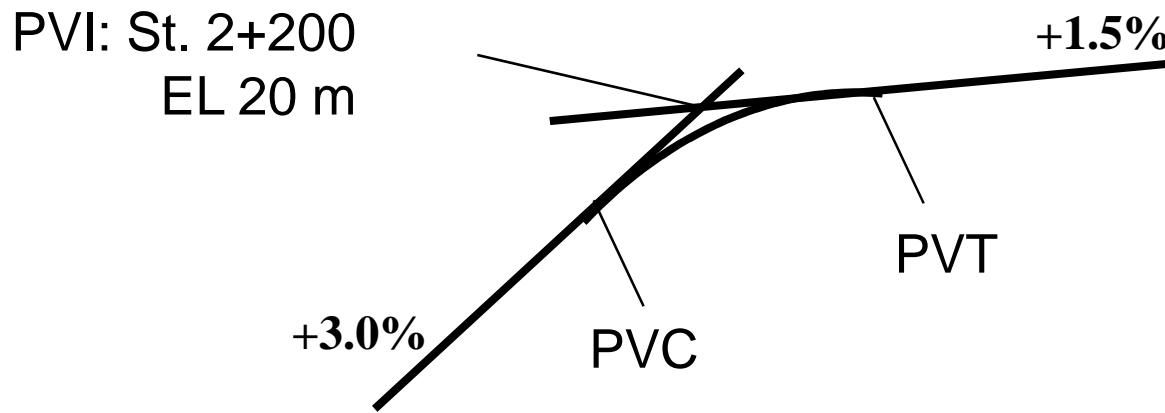
Example

A 400 m. equal tangent crest vertical curve has a PVC station of 1+000 at 59 m. elevation. The initial grade is 2.0 percent and the final grade is -4.5 percent. Determine the elevation and stationing of PVI, PVT, and the highest point of the curve.



Example

A vertical curve of 200m length connects the grades +3% and +1.5%. The station of PVI = 2+200 and the elevation of the PVI = 20.0m. Determine the station and elevation of the highest point on this curve.



K-Value

It defines the vertical curvature. It is the number of horizontal meter needed for a 1% change in slope

$$K = \frac{L}{A} \quad \text{high/low pt.} \Rightarrow x = K|G_1|$$

Crest Vertical Curves

- Assuming $L > SSD$...

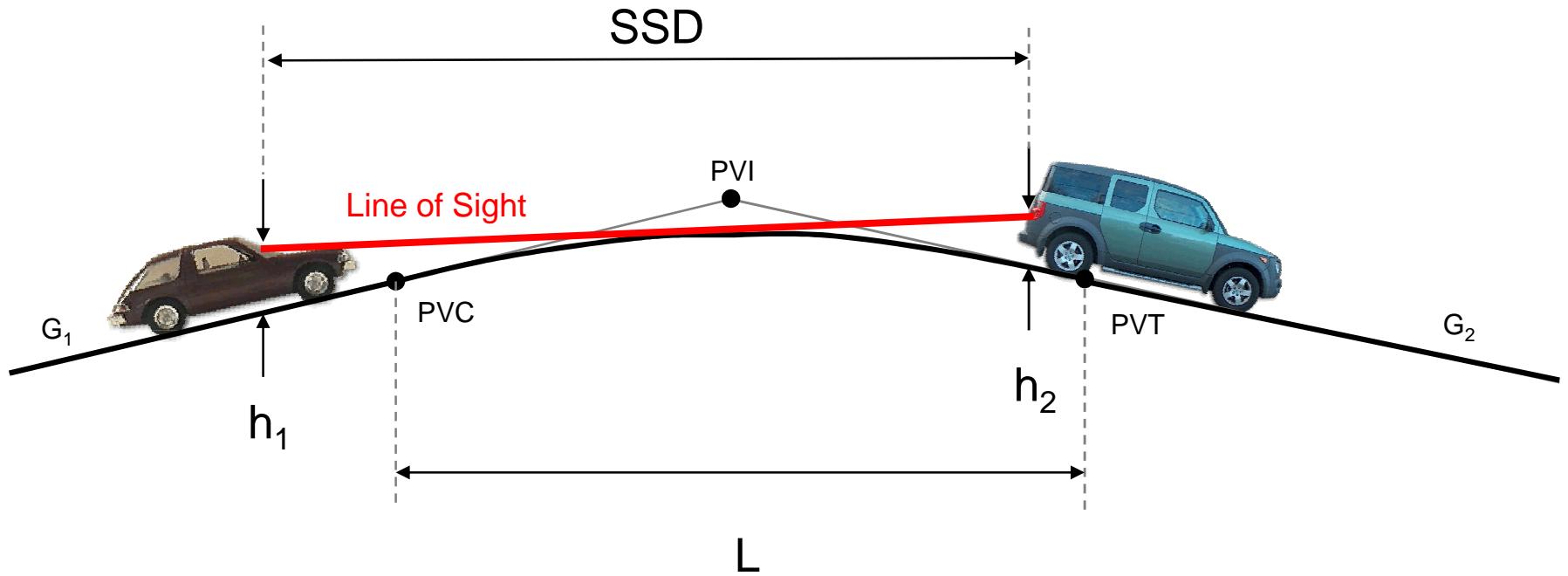
$$K = \frac{SSD^2}{400}$$

Sag Vertical Curves

- Assuming $L > SSD$...

$$K = \frac{SSD^2}{400 + 3.5SSD}$$

Crest Vertical Curves and S.S.D.



For SSD < L

$$L = \frac{A(SSD)^2}{100(\sqrt{2h_1} + \sqrt{2h_2})^2}$$

For SSD > L

$$L = 2(SSD) - \frac{200(\sqrt{h_1} + \sqrt{h_2})^2}{A}$$

Crest Vertical Curves based on SSD

- Assumptions for design
 - h_1 = driver's eye height = 1.05m
 - h_2 = tail light height = 0.15m
- Simplified Equations

For $SSD < L$

$$L = \frac{A(SSD)^2}{658}$$

For $SSD > L$

$$L = 2(SSD) - \frac{658}{A}$$

Crest Vertical Curves based on PSD

- Assumptions for design
 - h_1 = driver's eye height = 1.35m
 - h_2 = tail light height = 1.08m
- Simplified Equations

For PSD < L

$$L = \frac{A(PSD)^2}{864}$$

For PSD > L

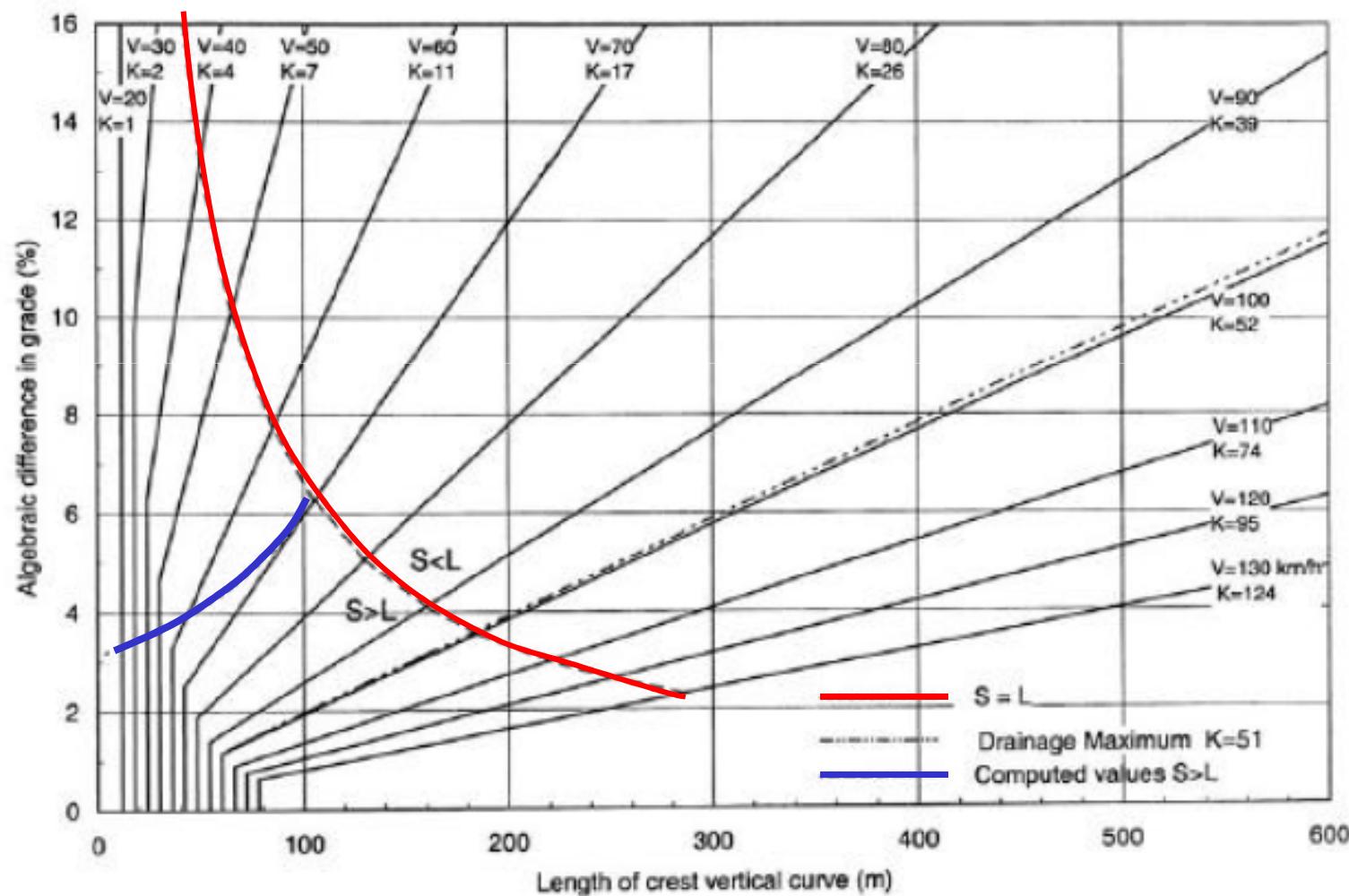
$$L = 2(PSD) - \frac{864}{A}$$

Design for Crest Curves Based on SSD

Design speed (km/h)	Metric		
	Stopping sight distance (m)	Calculated	Design
20	20	0.6	1
30	35	1.9	2
40	50	3.8	4
50	65	6.4	7
60	85	11.0	11
70	105	16.8	17
80	130	25.7	26
90	160	38.9	39
100	185	52.0	52
110	220	73.6	74
120	250	95.0	95
130	285	123.4	124

^a Rate of vertical curvature, K, is the length of curve per percent algebraic difference in intersecting grades (A). $K = L/A$

Design for Crest Curves based on PSD

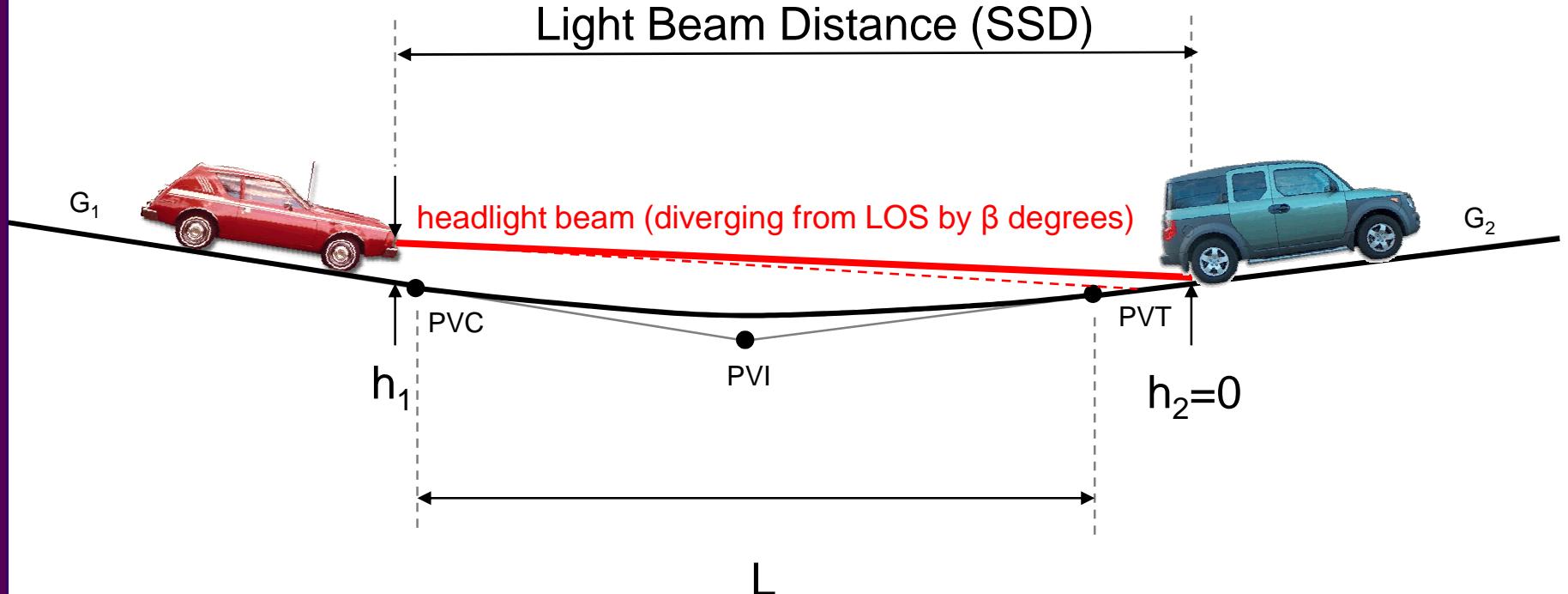


Design for Crest Vertical Based on PSD

Metric		
Design speed (km/h)	Passing sight distance (m)	Rate of vertical curvature, K* design
30	200	46
40	270	84
50	345	138
60	410	195
70	485	272
80	540	338
90	615	438
100	670	520
110	730	617
120	775	695
130	815	769

Note: *Rate of vertical curvature, K, is the length of curve per percent algebraic difference in intersecting grades (A). $K=L/A$

Sag Vertical Curves



For $SSD < L$

$$L = \frac{A(SSD)^2}{200(h_1 + S \tan \beta)}$$

For $SSD > L$

$$L = 2(SSD) - \frac{200(h_1 + (SSD) \tan \beta)}{A}$$

Headlight Sight Distance

- At night, the portion of highway that is visible to the driver is dependent on the position of the headlights and the direction of the light beam
- Headlights are assumed to be 60 cm and 1-degree upward divergence of the light beam from the longitudinal axis of the vehicle

Sag Vertical Curves

- **Assumptions for design**
 - h_1 = headlight height = 60 cm
 - β = 1 degree
- **Simplified Equations**

For $SSD < L$

$$L = \frac{A(SSD)^2}{120 + 3.5(SSD)}$$

For $SSD > L$

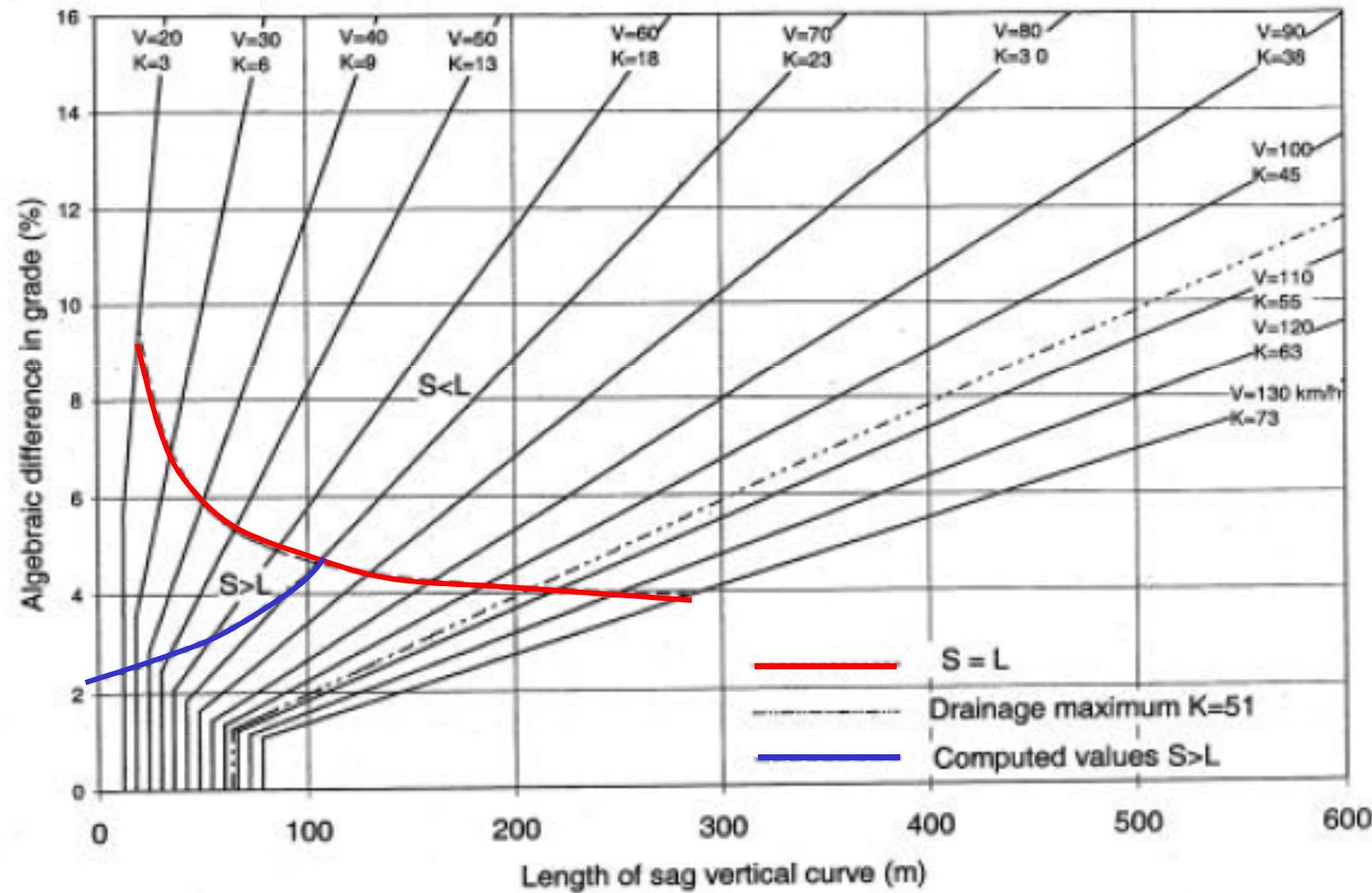
$$L = 2(SSD) - \left(\frac{120 + 3.5(SSD)}{A} \right)$$

Design Controls for Sag Vertical Curves

Metric			
Design speed (km/h)	Stopping sight distance (m)	Rate of vertical curvature, K^a	
		Calculated	Design
20	20	2.1	3
30	35	5.1	6
40	50	8.5	9
50	65	12.2	13
60	85	17.3	18
70	105	22.6	23
80	130	29.4	30
90	160	37.6	38
100	185	44.6	45
110	220	54.4	55
120	250	62.8	63
130	285	72.7	73

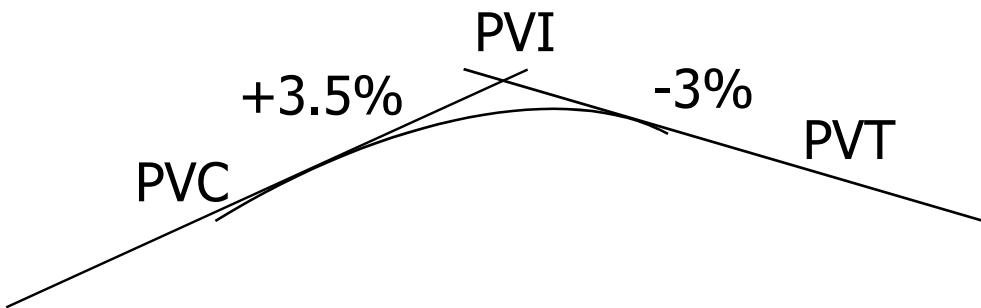
^a Rate of vertical curvature, K , is the length of curve (m) per percent algebraic difference intersecting grades (A). $K = L/A$

Design Controls for Sag Vertical Curves



Example

A +3.5% grade intersects a -3% grade at station 300+50 at an elevation 80.20m. Determine the length of the vertical connects the two grades based on SSD, then determine the station and elevation of PVC if the design speed = 80kph



Example

A vertical curve connecting two grades of +2% and -4% with length of 250m. What is the max. safe sped on this curve.

Example

A car is traveling at 50 kph in the country at night on a wet road through a 50 m long sag vertical curve. The entering grade is -2.4 percent and the exiting grade is 4.0 percent. A tree has fallen across the road at approximately the PVT. Assuming the driver cannot see the tree until it is lit by her headlights, is it reasonable to expect the driver to be able to stop before hitting the tree?

Example

Similar to previous example but for a crest curve.

A car is traveling at 50 kph in the country at night on a wet road through a 50 m long crest vertical curve. The entering grade is 3.0 percent and the exiting grade is -3.4 percent. A tree has fallen across the road at approximately the PVT. Is it reasonable to expect the driver to be able to stop before hitting the tree?

Example

A roadway is being designed using a 45 mph design speed. One section of the roadway must go up and over a small hill with an entering grade of 3.2 percent and an exiting grade of -2.0 percent. How long must the vertical curve be?

الحمد لله



Any Questions?